

# APRON: NUMERICAL PROGRAM ANALYSIS

<http://apron.cri.ensmp.fr/>

CRI/École des Mines — École Normale Supérieure — École Polytechnique — Vérimag/CNRS — INRIA

## Approach

- Static analysis by abstract interpretation
- Define an invariant at each program point (semantic fixpoint equations)
- Solve the equations using
  - abstract semantics (abstract properties)
  - extrapolation

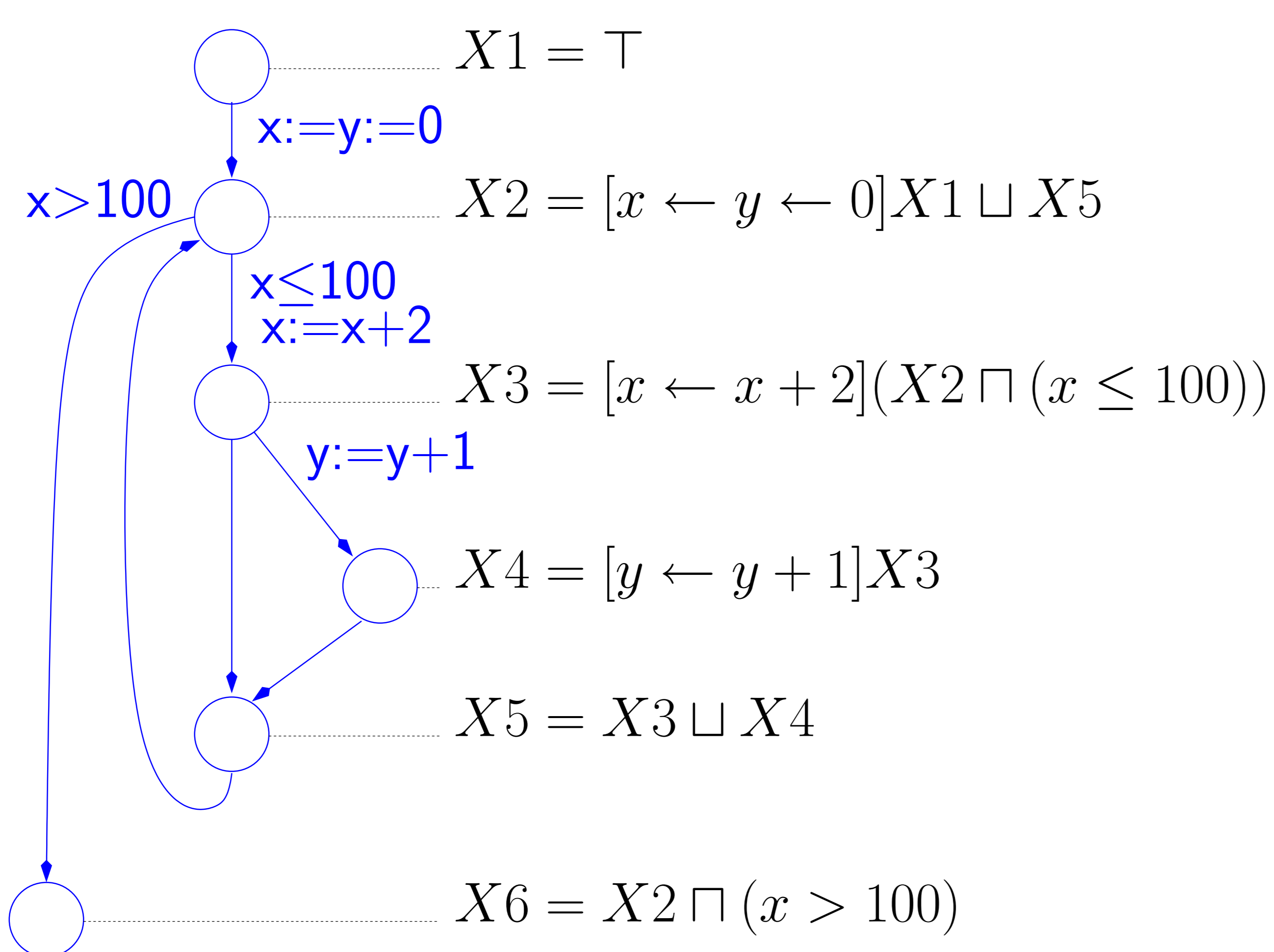
## Previous success stories

**Astrée** [ENS/CNRS]: Successful analysis of the A380 flight control software (700 KLoC).

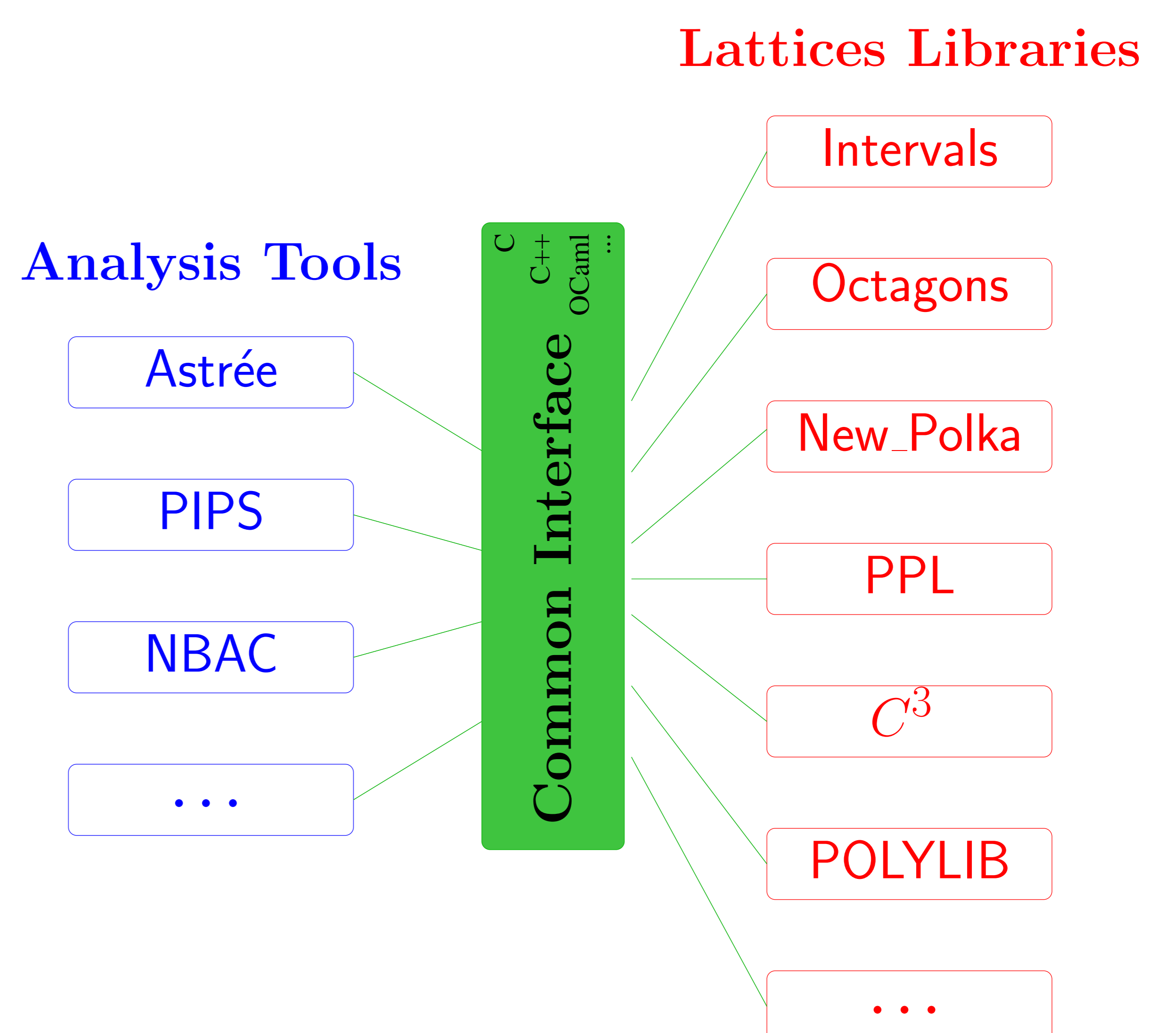
**Two companies:**

- PolySpace Technologies: Analysis of runtime errors.
- AbsInt Angewandte Informatik GmbH: Precise evaluation of worst case execution time.

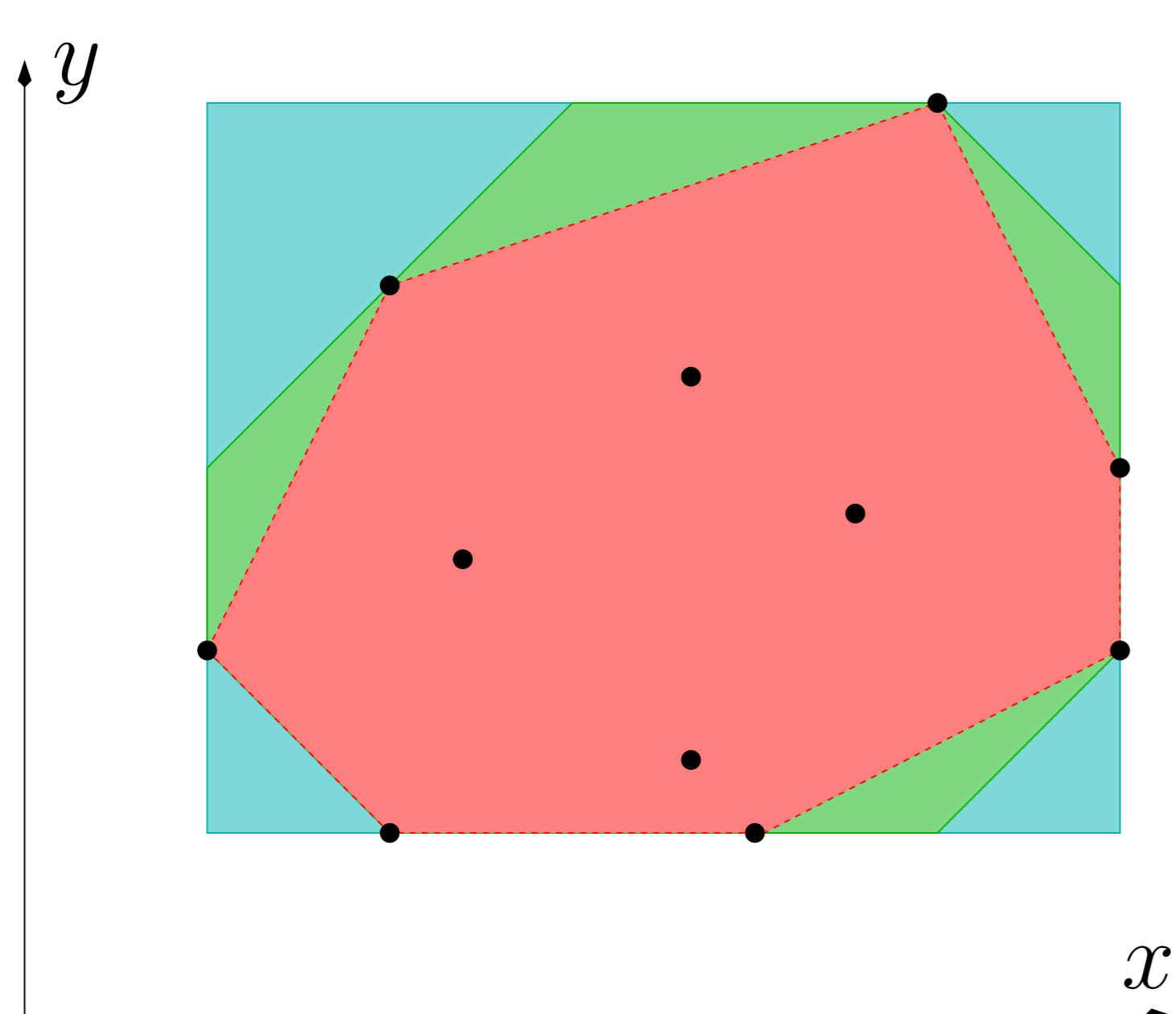
## Semantic equations



## Joint work: Common interface



## Numerical lattices



Intervals:  $x \in [1, 6]$   $y \in [1, 5]$

Octagons:  $x \in [1, 6], y \in [1, 5], (x + y) \in [3, 10], (x - y) \in [-2, 4]$

Polyhedra:  $1 \leq y \leq 2x \leq 12, 3 - y \leq x \leq 2 + 2y, 2x + y \leq 15$

- Representation, Normal form
- Emptiness decision
- Lattice operations ( $\sqcup, \sqcap, \sqsubseteq$ )
- Image by an assignment, guard, substitution
- Extrapolation operators

## Ongoing work

- **Relational analysis of floating-point computations:** discover sound bounds taking rounding into account.
- **Non-linear abstractions of domain-specific properties:** analysis of complex code patterns (e.g., digital filtering, slowly diverging geometric sequences).
- **Functional properties:** modular proofs that implementations of elementary numerical functions satisfy their specifications.
- **Parameterized abstractions:** symbolic analyzes parameterized by numerical domains (shape analysis, class invariants).
- **Automatic termination proofs** using semidefinite programming optimization.
- **Abstract acceleration on polyhedra:** compute, when possible, the exact limit of fixpoint iterations in the lattice of polyhedra.
- **DBMs with disequalities:** a new lattice for representing order and disequality ( $x \neq y$ ) relations between variables.

